Math 3450 - Homework # 1
Set Builder Notation

1. Find all the elements from the set $\{n \in \mathbb{Z} \mid 1 \leq n^2 \leq 100\}$.

   Solution: $-10, -9, 8, -7, -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

2. Let $X = \{x \in \mathbb{R} \mid x^2 + 1 = 0\}$. What set is X equal to?

   Solution: There are no real numbers x with $x^2 + 1 = 0$. Hence, $X = \emptyset$.

3. Find all the elements in the set $A = \{x \in \mathbb{N} \mid x^2 \leq 9\}$.

   Solution: Recall that $\mathbb{N} = \{1, 2, 3, 4, 5, 6, \ldots\}$. The only elements $x \in \mathbb{N}$ with $x^2 \leq 9$ are $x = 1$, $x = 2$, and $x = 3$. Hence $X = \{1, 2, 3\}$.

4. Let $S = \{1, 5, 7\}$ and $T = \{-1, 0, 10, 5\}$. Find all the elements in the set $X = \{a + b \mid a \in S, b \in T\}$.

   Solution:

$$\begin{aligned} X &= \{1 + (-1), 1 + 0, 1 + 10, 1 + 5, \\ &\quad 5 + (-1), 5 + 0, 5 + 10, 5 + 5, \\ &\quad 7 + (-1), 7 + 0, 7 + 10, 7 + 5\} \\ &= \{0, 1, 11, 6, 4, 5, 15, 10, 6, 7, 17, 12\} \\ &= \{0, 1, 4, 5, 6, 7, 10, 11, 12, 15, 17\} \end{aligned}$$

5. Let $S = \{1, 5, 7\}$. Find all the elements in the set $Y = \{a^2 \mid a \in S\}$.

   Solution: $Y = \{1^2, 5^2, 7^2\} = \{1, 25, 49\}$

6. List 5 elements from the set $S = \{2x - 3y \mid x, y \in \mathbb{Z}\}$.

   Solution: 5, 2, -1, -7, and -2 are all elements in S. This is because $5 = 2(1) - 3(1)$, $2 = 2(1) - 3(0)$, $-1 = 2(7) - 3(5)$, $-7 = 2(-2) - 3(1)$, and $-2 = 2(2) - 3(2)$.

7. Suppose that k is some fixed integer. List 10 elements from the set $S = \{xk \mid x \in \mathbb{Z}\}$.

   Solution: $k, 2k, 3k, 10k, 104k, -k, -7k, -81k, 1765k, -100k$ are all elements of S.

   Note that if $k = 0$, then these are all the same.

8. Suppose that r and s are two fixed integers. List 10 elements from the set $A = \{xr + ys \mid x, y \in \mathbb{Z}\}$.

   Solution: $r+s, 2r-5s, 3r+6s, 4r-10s, 5r = 5r+0s, s = 0r+s, 7r-109s, 8r+15s, 9r-163s, 10r+s$ are all elements of S.

9. Use set-builder notation to write the set of all positive odd numbers.

   Solution: Possible answer: $\{2k-1 \mid k \in \mathbb{N}\}$. This works because

$$\begin{aligned}\{2k-1 \mid k \in \mathbb{N}\} &= \{2(1)-1, 2(2)-1, 2(3)-1, 2(4)-1, \ldots\} \\ &= \{1, 3, 5, 7, \ldots\}.\end{aligned}$$